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Supersymmetric theories of dyons

Manu Pratap Singh
Department of Computer Science, Gurubul Kangri University,
Hardwar, India

and

B S Rajput
Department of Physics, Kumaun University,
Nainital-263 001, India

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Abstract : Constructing a suitable Lagrangian for non-Abelian dyons, the expression for dyonic mass has been derived and a connection of moduli space of monopoles with normalized bosonic and fermionic zero modes has been established. Supersymmetrizing this dyonic Lagrangian density, the study of supersymmetric dyons has been undertaken in $N = 2$ and $N = 4$ supersymmetric theories and it has been shown that the modification in supersymmetric algebra, in presence of central charges, leads to partial breaking of supersymmetries while the unbroken supersymmetry pairs the bosonic and fermionic zero modes.

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1. Introduction

Monopoles and dyons have become intrinsic parts of all current grand unified theories [1] with enormous potential importance in connection with their roles in catalyzing proton decay [2,3], the quark confinement problem of QCD [4,5] and RCD [6,7], and CP-violation in terms of non-zero vacuum angle [8]. We have worked out dyon-fermion dynamics [9] and demonstrated that the nature of the dyons is strongly perturbed by fermionic sector which couples with them. In most of supersymmetric theories [10–13] the spontaneous breaking of symmetry at mass scale $M_x \sim 10^{15}$ GeV, leads to the presence of monopoles and dyons. If supersymmetry breaks at scale much less than M_x, the monopoles and dyons must form supermultiplets of approximately degenerate boson and fermion states [14–16]. In supersymmetric GUT's, the monopole ground state, in general, carries fractional electric charge as well as color hyper-charge [17]. This is manifestation of fermion fractionalization [18] with the axial anomaly effect [2] properly taken into account. Supersymmetry provides the first realistic testing ground for the idea of fermion fractionalization and induced fermionic charge on a monopole. As such the occurrence of fractionally charged dyons along with their superpartners for each charge state, due to symmetry, leads to the possibility of observation of dyons. The fractional charge of dyons arises for the Higgs-boson mass case (and not for Dirac mass case) and the non-trivial topology of the background Higgs fields leads to Jackiw-Rebbi zero modes [18] even in the supersymmetric theories where the role of fermionic zero modes is manifest only after quantization. Dynamics of $N = 1$ supersymmetric gauge theories of monopoles and dyons has been much explored partly because of phenomenological interest and the recent results [19–21] have emerged about their strong coupling behaviour. But in general, it is very difficult to obtain the explicit form of Jackiw-Rebbi zero modes in supersymmetric theories, and the dual nature of the fermionic zero modes in $N = 2$ super Yang-Mills theories leads to several difficulties in dealing with the problem of monopoles, dyons and dyonic supermultiplets.

In the present paper, we have analysed the extended structure of non-Abelian dyons and obtained the expressions for dyonic mass and electric and magnetic fields in the interior region of dyons and showed that when collective coordinates of monopole are time dependent, it acquires momentum and electric charge and becomes a moving dyon. Constructing suitable moduli space of monopoles, its connection with normalized bosonic and fermionic zero modes has been established. Supersymmetrizing the dyonic Lagrangian density by introducing fermionic spinors and pseudoscalars into it, the study of supersymmetric dyons has been undertaken in $N = 2$ and $N = 4$ supersymmetric theories and it has been shown that the dyonic mass formula survives quantization when supersymmetric charge algebra is saturated in a particular fashion. It has also been shown that the modification in supersymmetric algebra, in the presence of central charges, leads to partially breaking of supersymmetry and the unbroken supersymmetry pairs the bosonic zero modes with fermionic zero modes in both $N = 2$ and $N = 4$ supersymmetric theories of dyons.

2. Fields associated with non-Abelian dyons

A general non-Abelian gauge theory of dyons consists of usual four-space (external) and n-dimensional internal group space, where the field associated with dyons has n-fold internal multiplicity and the multiplets of gauge field transform as a basis of adjoint representation of n-dimensional non-Abelian gauge symmetry group. Choosing the internal gauge group as SU (2), the generalized dyonic field tensor may be constructed as

$$\vec{G}_{\mu\nu} = G_{\mu\nu}{}^{a} T_{a} \tag{2.1}$$

with the generalized four-potential defined as

$$\vec{V}_{\mu} = V_{\mu}{}^{a} T_{a}. \tag{2.2}$$

where the vector sign $\rightarrow$ is denoted in the internal group space; $\mu, \nu = 0, 1, 2, 3$ represent the degrees of freedom in the external space and matrices T_a are the infinitesimal generators of group SU(2). We may connect $\vec{G}_{\mu\nu}$ and $\vec{V}_\mu$ in the following manner:

$$\vec{G}_{\mu\nu} = \partial_\nu \vec{V}_\mu - \partial_\mu \vec{V}_\nu + |q| [\vec{V}_\mu, \vec{V}_\nu] \tag{2.3}$$

which may also be written as

$$G^a_{\mu\nu} = \partial_\nu V^a_\mu - \partial_\mu V^a_\nu + |q| \epsilon^{abc} B_{\mu b} V_{\nu c} \tag{2.4}$$

where $$q = e - ig \tag{2.5}$$

is the dyonic generalized charge with e and g as electric and magnetic constituents.

A suitable Lagrangian density of a spontaneously broken non-Abelian gauge theory, yielding classical dyonic solutions, may be constructed as [22]

$$L = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + (D_\mu \phi)^a (D^\mu \phi)_a - V(\phi), \tag{2.6}$$

where $$(D_\mu \phi)^a = \partial_\mu \phi^a + |q| \epsilon^{abc} V_{\mu b} \phi_c \tag{2.7}$$

and $$V(\phi) = \frac{1}{4}(\phi^a \phi_a)^2 - \frac{1}{2} v^2 (\phi^a \phi_a) \tag{2.8}$$

with $v = < \phi >$.

Using this Lagrangian density, we have computed [22] the electric and magnetic fields and the resulting angular momentum of dyons by imposing the following ansatz [23];

$$\begin{aligned} V_{ia} &= \epsilon_{aij} (\hat{r})^j [K(r) - 1] / |q| r, \\ V_{oa} &= (\hat{r})_a J(r) / |q| r, \\ \phi_a &= (\hat{r})_a H(r) / |q| r, \end{aligned} \tag{2.9}$$

which satisfy the following equations

$$\begin{aligned} r^2 H''(r) &= 2 H K^2, \\ r^2 J''(r) &= 2 J K^2, \\ r^2 K''(r) &= K(K^2 - 1) + K(H^2 - J^2). \end{aligned} \tag{2.10}$$

In Prasad-Sommerfield limit [24]

$$V(\phi) = 0$$

but $$v = < \phi > \neq 0.$$

In this limit, the dyons have lowest possible energy for given electric and magnetic charges e and g respectively. Then we get the following expression for dyonic mass

$$M = v(e^2 + g^2)^{1/2} = v|q|, \tag{2.11}$$

when the electric and magnetic fields associated with dyons obey the first order equations

$$E_i^a = G_{oi}^a = (D_i\phi)^a \sin\alpha, \tag{2.12}$$

$$B_i^a = \frac{1}{2}\epsilon_{ijk}\, G^{jka} = (D_i\phi)^a \cos\alpha, \tag{2.12a}$$

and $\quad (D_0\phi)^a = 0,$

where $\quad \alpha = \tan^{-1} e/g.$

Using Gauss's law and these expressions for fields, we have

$$e = \frac{1}{v}\int d^3x\partial_i\left(\phi^a G_{oi}^a\right)$$

and

$$g = \frac{1}{2}\cdot\frac{1}{v}\int d^3x\,\epsilon_{ijk}\;\partial_i\left(\phi^a G_{ojk}^a\right). \tag{2.13}$$

For the case of pure monopole $\alpha = o$ and $V^{oa} = o$ and then eq. (2.12a) reduces to the Bogomol nyi equation [25]

$$B_i = D_i\,\phi. \tag{2.14}$$

In such a case, eq. (2.8) gives the static energy which follows the Bogomol'nyi bound

$$V \geq |k|, \tag{2.15}$$

where k is the monopole number given by

$$k = \int d^3x\,\partial_i tr(B_i\,\phi). \tag{2.15a}$$

The condition (2.14) does not allow static dyonic solutions but in this case the dyons emerge as time dependent solutions [26] and the ansatz given by eqs. (2.9) reduces to Prasad-Sommerfield condition and then the solutions of Bogomol'nyi eq. (2.14) give BPS monopole as a static spherically symmetric soliton with smooth field and finite mass. The Bogomol'nyi eq. (2.14) are equivalent to the self duality equations of pure Yang-Mills in R^4-space restricted to be translationally invariant in one direction [27]. Let us construct a connection on R^4 that is translationally invariant in the x^4-direction *via*

$$E_i^a = V_i^a;\quad W_4^a = \phi^a. \tag{2.16}$$

If $\vec{F}_{\mu\nu}$ is the field strength corresponding to $\vec{W}_\mu$ then the self duality equations

$$\vec{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\,\vec{F}^{\rho\sigma} \tag{2.17}$$

are equivalent to Bogomol'nyi eq. (2.14).

Introducing covariant derivative D_μ on R^4 with $\vec{V}_\mu$ replaced by $\vec{W}_\mu$ in eq. (2.7) for $e = 0$, we may write Gauss's law as

$$D_u\,\dot{\vec{W}}_\mu = 0 \tag{2.18}$$

and infinitesimal gauge transformations as

$$\delta \vec{W}_\mu(x) = D_\mu \vec{\Lambda}, \tag{2.19}$$

where gauge parameter $\Lambda(x)$ is restricted to be independent of x^4. It follows from these relations that BPS monopoles are topological solitons in a Yang-Mills gauge theory in three space dimensions. Such a monopole has four collective coordinates which include three position coordinates and a phase angle. When these four coordinates are time dependent, the monopole acquires momentum and electric charge and it becomes a moving dyon.

It is well known that there exists a zero frequency solution (a zero mode) for every generator of the broken symmetry that does not annihilate the classical solutions of eq. (2.10). The number of zero modes in the Prasad-Sommerfield limit is a multiple of four. To be more specific about these zero modes, let us define moduli space M_k of k monopoles as the complete set of solutions of Bogomol'nyi eq. (2.14) within a topological class k. The natural set of coordinates for M_k are arbitrary parameters or moduli $\{X^\alpha, \alpha = 1 \ldots\ldots \dim (M_k)\}$ that determine the gauge equivalence class of solutions $[\vec{W}_\mu(x, X)]$. Besides the Gauss law (2.18), the tangent vectors to M_k must also obey the linearized Bogomol'nyi equations

$$D_\mu \dot{\vec{W}}_\nu - D_\nu \dot{\vec{W}}_\mu = \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} D_\rho \dot{\vec{W}}_\sigma. \tag{2.20}$$

There is the following close connection between zero modes in the fluctuations about a particular monopole background and tangent vectors to M_k;

$$\dot{W}_\mu = \dot{X}^\alpha \delta_\alpha \vec{W}_\mu,$$

where
$$D_\mu \delta_\alpha \vec{W}_\nu - D_\nu \delta_\alpha \vec{W}_\mu = \frac{1}{2} \epsilon_{\mu\nu\sigma\rho} D_\rho \delta_\alpha W_\sigma \tag{2.21}$$

and
$$D_\mu \delta_\alpha W_\mu = 0. \tag{2.22}$$

Eq. (2.21) is the precise statement that $\delta_\alpha W_\mu$ is a zero mode and eq. (2.22) states that it is orthogonal to gauge modes. For BPS system, all zero modes are normalizable and there is a one to one correspondence between normalizable zero modes and moduli M_k which is hyper-Kähler manifold.

The nature of dyons is strongly perturbed by fermionic sector which couples with them. In quantum theory, there are fermionic zero modes in addition to bosonic zero modes in the fluctuations about monopole solutions. These fermionic zero modes [18] are time independent C-number solutions of Dirac equation in the presence of monopoles. In other words, the Dirac operator coupled to a t'Hooft-Polyakov monopole has zero modes. For dyons also the self conjugate zero energy solutions continue to exist for both isospinor and isovector fermions. The fractional charge of dyons arises for the Higg boson mass case and the non-trivial topology of the background Higgs field leads to Jackiw-Rebbi zero modes.

For BPS monopoles, the absence of static interactions implies that the collective coordinate Lagrangian consists only of a kinetic energy term

$$L = \frac{1}{2} g_{\alpha\beta} \dot{X}^{\alpha} \dot{X}^{\beta} \ , \tag{2.23}$$

where X are collective coordinates and $g_{\alpha\beta}$ is the metric on the moduli space spanned by X^{α}. The fields W_i^a and ϕ_i^a, given by eq. (2.16), depend on these collective coordinates as well as on R^4. The moduli space metric can be obtained directly from the zero modes provided these modes satisfy the background gauge condition

$$D_{\mu} \delta_{\alpha} W_{\mu}^{a} = 0. \tag{2.24}$$

For this condition, it is necessary to introduce an infinitesimal gauge transformation $\vec{\epsilon}_{\alpha}$ to zero modes so that these zero modes corresponding to a collective coordinate X^{α} will take the form

$$\delta_{\alpha} \vec{W}_{\mu} = \partial_{\alpha} \vec{W}_{\mu} - D_{\mu} \vec{\epsilon}_{\alpha}, \tag{2.25}$$

where $\quad \partial_{\alpha} = \dfrac{\partial}{\partial X^{\alpha}}$

and the gauge parameters $\epsilon'_{\alpha} = \epsilon'_{\alpha} (x, X^{\alpha})$ satisfy the condition (2.22). Then the moduli space metric is given by

$$g_{\alpha\beta} = \int d^3x \, tr\left(\delta_{\alpha} W_{\mu} \delta_{\beta} W^{\mu}\right). \tag{2.26}$$

The gauge parameters of eq. (2.25) can be viewed as defining a natural connection on M_k with the covariant derivative

$$\vec{\nabla}_{\alpha} = \partial_{\alpha} + [\vec{\epsilon}_{\alpha}, \ ]. \tag{2.27}$$

Then we have bosonic zero modes as

$$\delta_{\alpha} \vec{W}_{\mu}{}^{a} = [\vec{\nabla}_{\alpha}, D_{\mu}]. \tag{2.28}$$

A low energy ansatz for the fields is obtained by demanding that the only time dependence is *via* the collective coordinates :

$$W_{\mu}^{a}(x, t) = W_{\mu}^{a}(x, X^{\alpha}(t)), \ \ V_0 = \dot{X}^{\alpha} \epsilon_{\alpha} \tag{2.29}$$

where V_0 term is necessary to ensure that motion is constrained to be orthogonal to gauge transformations. While working in the gauge $V_0 = 0$ (for Pure monopoles), one should really perform a gauge transformation on the ansatz to stay in this gauge.

3. Supersymmetric dyons

It has been shown [19] that the monopole states that are connected by Jackiw-Rebbi zero mode operators must be embedded in to the fundamental multiplets of $N = 1$

supersymmetric Georgi-Glassow model and these modes exactly coincide with the supersymmetric zero modes [28]. In fact, the Bogomol'nyi bound (2.15) can be deduced from the presence of certain topological charges that appear is supersymmetric algebra. In the following subsections, we shall construct the dyonic multiplets in $N = 2$ and $N = 4$ supersymmetric theories :

(a) Supersymmetric dyons in $N = 2$ theory :

The simplest four-dimensional supersymmetric model in which boundary terms enter as central charge is the following $N = 2$ supersymmetric version of the dyonic Lagrangian given by eq. (2.6);

$$\begin{aligned} L = & -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + \frac{1}{2} (D_\mu \phi)^a (D^\mu \phi)_a \\ & + \frac{1}{2} (D_\mu P)^a (D^\mu P)_a - V(\phi, P) + i \overline{\psi}^a \gamma^\mu (D_\mu \psi)_a \\ & - |q| \epsilon_{abc} \overline{\psi}^a \gamma_5 \psi^c P^b \\ & - i |q| \epsilon_{abc} \overline{\psi}^a \psi^b \phi^c, \end{aligned} \tag{3.1}$$

where

$$\begin{aligned} V(\phi, P) = & \frac{1}{4}\left(|q| \epsilon_{abc} \phi^b \phi^c\right)^2 + \frac{1}{4}\left(|q| \epsilon_{abc} P^b P^c\right)^2 \\ & - \frac{1}{2}\left(|q| \epsilon_{abc} \phi^b P^c\right)^2, \end{aligned} \tag{3.1a}$$

ψ is Dirac spinor, ϕ is a scalar field and P is a pseudoscalar (all these fields are in the adjoint representation of the gauge group). This Lagrangian of SU (2) Weiss-Zumino model obviously describes the dyon-isovector-fermion system since the dyonic solutions which arose naturally from the Lagrangian (2.6) will also exist is its supersymmetric version (3.1) because non-trivial topology of the dyonic fields supports their stability in both the theories.

Under the supersymmetric transformations, the fields included in Lagrangian (3.1) transform as follows :

$$\begin{aligned} \delta V^a_\mu &= i\left(\overline{\alpha} \gamma_\mu \psi^a - \overline{\psi}^a \gamma_\mu \hat{\alpha}\right), \\ \delta \phi^a &= i\left(\overline{\psi}^a \hat{\alpha} - \bar{\hat{\alpha}} \psi^a\right), \\ \delta P^a &= \overline{\psi}^a \gamma_5 \hat{\alpha} - \bar{\hat{\alpha}} \gamma_5 \psi^a, \\ \delta \psi^a &= \left(\sigma_{\mu\nu} G^{\mu\nu a} + i |q| \epsilon^{abc} \gamma_5 P_b \phi_c + i \gamma^\mu D_\mu P^a \gamma_5 - \gamma^\mu D_\mu \phi^a\right) \hat{\alpha}, \end{aligned} \tag{3.2}$$

where $\hat{\alpha}$ is a constant anticommuting Dirac-Spinor. An important property of this model is that the vacuum energy $V(\phi, P)$ is independent of values of ϕ and P in certain directions in field space. As long as ϕ and P commute, the vacuum energy is classically zero. ϕ and /or P may have non-zero vacuum expectation values, spontaneously breaking some of the gauge symmetries. Since the potential for Higg's fields has flat directions corresponding to $[\phi, P] = 0$, by performing a chiral rotation we can assume that $<P> = 0$. As such the potential term $V(\phi, P)$ in Lagrangian (3.1) preserves chiral symmetry but breaks the supersymmetry yeilding dyonic solutions (2.12) and (2.12a) for

$$\begin{aligned} \langle 0|\phi^a|0\rangle &= \langle\phi\rangle = v, \\ \langle 0|P^a|0\rangle &= \langle P\rangle = 0. \end{aligned} \tag{3.3}$$

Thus, the original massless gauge multiplets split into two different supersymmetric multiplets, one of them is still massless and contains the fields V_μ^3, ϕ^3, P^3 and ψ^3 while the other one gets non-zero mass given by eq. (2.11) and contains the fields $V_\mu^\pm$, $\phi^\pm$ and $\psi^\pm$.

In the general case, for the Lagrangian density (3.1), the eq. (2.12) of the fields associated with dyons are modified into the following form :

$$\begin{aligned} (E_i\phi) &= \phi_a G_{0i}^a + \frac{1}{2} P_a \in_{ijk} G^{jka}, \\ (B_i\phi) &= \frac{1}{2}\phi_a \in_{ijk} G^{ija} + P_a G_{0i}^a. \end{aligned} \tag{3.4}$$

The supersymmetric current associated with Lagrangian (3.1) may be derived in the following form :

$$\begin{aligned} S_\mu &= \sigma^{\alpha\beta} G_{\alpha\beta}^a \gamma_\mu \psi^a + (D_\alpha\phi)^a \gamma^\alpha\gamma_\mu\psi^a \\ &\quad + (D_\alpha P)^a \gamma^\alpha \overline{\psi}^a - |q|\gamma_\mu\gamma_5 \in_{abc} \phi^b P_c \psi^a, \end{aligned} \tag{3.5}$$

which leads to the following expressions for electric and magnetic charges

$$\begin{aligned} Q_E &= \int d^3x\, \partial_i (E_i\phi), \\ Q_M &= \int d^3x\, \partial_i (B_i\phi) = k, \end{aligned} \tag{3.6}$$

with E_i and B_i given by eq. (3.4). This Q_E arises from the components of six-dimensional momentum in extra dimensions [14] while the magnetic charge Q_M has a topological origin. These central charges are nonvanishing only if the vacuum expectation values $<\phi>$ or $<P>$ is non-zero. Presence of these central charges in the algebra implies that

$$M^2 \geq Q_E^2 + Q_M^2. \tag{3.7}$$

In the special case given by eq. (3.3), the non-zero expectation value $<\phi> = v$, spontaneously breaks SU (2) down to U (1) and then eqs. (3.6) and (2.13) give

$$Q_E = v\, e$$

and $$Q_M = v\, g \tag{3.8}$$

Then eq. (3.7) reduces to

$$M \geq v\,|q| \tag{3.9}$$

which is saturated in the form of eq. (2.11), at classical level, for all states in the theory. When this saturation occurs, all the states in the theory (photon, Higgs particles, fermions, monopoles and dyons) satisfy the mass formula (2.11). The bound on mass in eq. (3.7) and (3.9) is the consequence of the presence of central charges [29]. The mass formula given by eq. (3.9), or eq. (2.11) in the saturated form, survives quantization by requiring that the supersymmetric charge algebra is saturated, in a particular fashion, involving central charges which can arise as surface terms when spontaneous symmetry breaking occurs [30]. As such in the present theory there are no quantum corrections to the classical mass spectrum given by eq. (2.11).

In the presence of central charges Q_E and Q_M given by eq. (3.6), the supersymmetric algebra is modified into the following form

$$\left\{\overline{Q}, Q\right\} = 2\gamma_\mu P^\mu - 2Q_E - 2i\gamma_5 Q_M. \tag{3.10}$$

This modification in algebra leads to partial breaking of supersymmetry [14] and the unbroken supersymmetry pairs the bosonic zero modes with fermionic zero modes (which can be interpreted as the Goldstone modes of the broken supersymmetry). There is really one to one correspondence between bosonic and fermionic zero modes.

The fermionic zero modes are time independent C-number solutions to the following Dirac equation in the presence of dyons;

$$\gamma^i D_i \psi^a - |q| \in^{abc} \phi_b \psi_c = 0. \tag{3.11}$$

The broken supersymmetry with C-number parameters generates two independent zero modes satisfying

$$\Gamma_5 \psi^a = -\psi^a,$$

where $\quad \Gamma_5 = -i\gamma_0\gamma_5.$

In the one dyon sector, these are the only fermionic zero modes while in multi-dyon sector, there are other zero modes also. According to Callias index theorem [31], in the k-dyon sector there are 2 k fermionic zero modes.

Let us introduce following euclidean gamma matrices

$$\Gamma_i = \gamma_0\gamma_i; \quad \Gamma_4 = \gamma_0; \quad \Gamma_5 = \Gamma_1\Gamma_2\Gamma_3\Gamma_4 \tag{3.12}$$

satisfying $\quad \left\{\Gamma_\mu, \Gamma_\nu\right\} = 2\delta_{\mu\nu}.$

Then we impose the following supersymmetric invariant restrictions on the equation of motion;

$$\Gamma_5\psi^a = -\psi^a; \quad V_0^a = -P^a \tag{3.13}$$

and demand that fields are time independent. As such, equations of motions consist of self duality equations (2.17) and the equations

$$\Gamma_\mu D_\mu \psi^a = 0; \quad D_\mu D_\mu P^a = i|q| \in^{abc} \psi_b^\dagger \psi_c \tag{3.14}$$

with the unbroken supersymmetric transformations given by

$$
\begin{aligned}
\delta W_\mu^a &= i\hat{\alpha}_+^\dagger \Gamma_\mu \psi^a - i\psi^{\dagger a}\Gamma_\mu \hat{\alpha}_+, \\
\delta\psi^a &= -2\Gamma_\mu D_\mu P^a \alpha_+, \\
\delta P^a &= 0.
\end{aligned}
\tag{3.15}
$$

Eqs. (2.17) and (3.14) are covariant under these transformations if $\hat{\alpha}_+$ is a Grassmann odd spinor. On the other hand if $\hat{\alpha}_+ = \varepsilon_+$ is a c-number spinor, then Dirac equation in eqs. (3.14) and (2.17) is not covariant while other two equations remain covariant. Thus for each fermionic zero mode (3.13) there is a bosonic zero mode

$$
\delta W_\mu^a = i\varepsilon_+^\dagger \Gamma_\mu \psi^a - i\psi^{a\dagger}\Gamma_\mu \varepsilon_+ \tag{3.16}
$$

which also satisfies eq. (2.21).

It may readily be shown [14] that

$$
\hat{\alpha}_\pm = 1/2\,[1 \pm \Gamma_5]\,\hat{\alpha}, \tag{3.17}
$$

where $\hat{\alpha}$ has been introduced through eq. (3.2). Here $\hat{\alpha}_+$ are the parameters of the unbroken symmetry and $\hat{\alpha}_-$ are the parameters of the broken symmetry. This result shows that the dyonic configuration breaks only half of the supersymmetry of the $N = 2$ theory. Thus generically, the solutions, satisfying the Bogomol'nyi bound, breaks only half of the supersymmetry in $N = 2$ theory and the unbroken supersymmetry pairs bosonic zero modes with fermionic zero modes. This partial breaking of the supersymmetry is a generic feature of the supersymmetric field theories admitting topologically non-trivial solutions This is a consequence of the fact that the algebra of supersymmetric charges is modified into the form given by eq. (3.10) by topological charges. As the result of this partial breaking of supersymmetry, the effective action governing the low energy dynamics of the dyons of $N = 2$ supersymmetry theory is given by $N = 4$ supersymmetric quantum mechanics. Furthermore, in $N = 2$ theory electrons and monopoles have different Lorentz quantum numbers (electrons are in a supersymmetric multiplet with spin ≤ 1, while the monopoles have spin $\leq 1/2$) and hence the electric-magnetic duality is not possessed in $N = 2$ theory.

(b) Supersymmetric dyons in $N = 4$ theory:

Osborn [30] obtained central charges in supersymmetric algebra of $N = 4$ supersymmetric gauge theory and demonstrated that when spontaneous symmetry breaking is imposed, the spins of topological monopole states become identical to those of massive elementary particles. Let us undertake the study of dyonic supersymmetric states in $N = 4$ Y–M theory by generalizing the Lagrangian density of eq. (3.1) into the following form:

$$L = \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + \frac{1}{2} (D_\mu \phi_i)^a (D^\mu \phi_i)_a$$
$$+ \frac{1}{2} (D_\mu P_i)^a (D^\mu P_i)_a - V(\phi, P)$$
$$+ i \overline{\psi}^a \gamma^\mu (D_\mu \psi)_a - i|q| \epsilon_{abc} \overline{\psi}^a \alpha^i \phi^b_i \psi^c$$
$$- |q| \epsilon_{abc} \overline{\psi}^a \gamma_5 \beta^j P^b_j \psi^c \tag{3.18}$$

where all fields are in adjoint representation of SU (2), $V(\phi, P)$ is given by

$$V(\phi, P) = \frac{1}{4}\left(|q| \epsilon_{abc} \phi^b_i \phi^c_j\right)^2 + \frac{1}{4}\left(|q| \epsilon_{abc} P^b_i P^c_j\right)^2$$
$$- \frac{1}{2}\left(|q| \epsilon_{abc} \phi^b_i P^c_i\right)^2, \tag{3.19}$$

and the 4 × 4 matrices α^i and β^j satisfy the following relations :

$$[\alpha^i, \alpha^j] = S^{ij} = -2 \epsilon^{ijk} \alpha_k,$$
$$[\beta^i, \beta^j] = V^{ij} = -2 \epsilon^{ijk} \beta_k,$$
$$\{\alpha^i, \beta^j\} = U^{ij} = \alpha^i \beta^j + \beta^j \alpha^i = -U^{ji},$$
$$[\alpha^i, \beta^j] = 0; \quad \{\alpha^i, \alpha^j\} = -2\delta^{ij},$$
$$\{\beta^i, \beta^j\} = -2\delta^{ij}. \tag{3.20}$$

The metric used is eqs. (3.18), (3.19) and (3.20) is $g_{ij} = -\delta_{ij}$ and $g_{\infty} = 1$.

The Lagrangian (3.18) is invariant under the following N = 4 supersymmetric transformations :

$$\delta V^a_\mu = i\left(\bar{\hat{\alpha}} \gamma_\mu \psi^a - \overline{\psi}^a \gamma_\mu \hat{\alpha}\right), \tag{3.21a}$$

$$\delta \phi^a_j = i\left(\alpha_j \overline{\psi}^a \hat{\alpha} - \alpha_j \hat{\alpha} \psi^a\right), \tag{3.21b}$$

$$\delta P^a_j = \alpha_j\left(\overline{\psi}^a \gamma_5 \overline{\alpha} - \bar{\hat{\alpha}} \gamma_5 \psi^a\right), \tag{3.21c}$$

$$\delta \psi^a = \Big[\sigma_{\mu\nu} G^{\mu\nu a} + i|q| \alpha^i \beta^j \epsilon^{abc} \gamma_5 P_b \phi_c$$
$$+ i\gamma^\mu D_\mu\left(\beta^j P^a_j\right)\gamma_5 - \gamma^\mu D_\mu\left(\alpha^i \phi^a_i\right)$$
$$+ \frac{1}{2} |q| \epsilon_{ijk} \epsilon^{abc} \left(\alpha^k \phi^i_b \phi^j_c + \beta^k P^i_b P^j_c\right)$$
$$- i|q| \epsilon^{abc} \alpha^i \beta^j (\phi_i)_a (P_j)_b \gamma_5\Big] \hat{\alpha}, \tag{3.21d}$$

where $\hat{\alpha}$ is a constant, anticommuting Majorana spinor.

The equation of motion for the gauge field is

$$D^{\nu}G^{a}_{\mu\nu} + i|q|\in^{abc}(\phi_i)_b(D_\mu\phi_i)_c + i|q|\in^{abc}(P_j)_b(D_\mu P_j)_c$$
$$-\frac{1}{2}|q|\in^{abc}(\overline{\psi})_b\gamma_\mu\psi_c = 0. \tag{3.22}$$

In Prasad-Sommerfield limit [24], we have

$$V(\phi, P) = 0,$$

but $$\langle 0|\Phi|0\rangle = v \neq 0, \tag{3.23}$$

where $$\Phi^a = A_i\phi^a_i + B_i P^b_j \tag{3.24}$$

and $$A_i A_i + B_j B_j = 1$$

with A_i and B_i as constants. Then zero order solution of eq. (3.22) is

$$E^a_i = (D_i\Phi)^a \sin\alpha,$$
$$B^a_i = (D_i\Phi)^a\cos\alpha = \frac{1}{2}\in_{ijk} G^{jka} \tag{3.25}$$

and $$(D_0\Phi)^a = 0.$$

For pure static monopole, $\alpha = 0$ and hence eq. (3.25) reduce to

$$E^a_i = 0, \quad V^a_0 = 0$$

and $$B^a_i = (D_i\Phi)^a \tag{3.26}$$

and then eq. (3.21d) reduces to

$$\delta\psi^a = \sigma_i B^a_i(1+\hat{P})\hat{\alpha} \tag{3.27}$$

where $$\sigma_i = \in_{ijk}\sigma^{jk} = -\gamma^i\gamma^0\gamma_5 \tag{3.28}$$

are Pauli marices and

$$\hat{P} = \hat{P}^{\dagger} = \gamma^0\gamma_5\alpha^j A_i + i\gamma^0\beta^j B_j \tag{3.29}$$

is a projection matrix.

The solution given by eq. (3.27) has broken half of the supersymmetries for

$$P\hat{\alpha} = -\hat{\alpha}$$

It may readily be shown [30] that relations (3.6) for central charges and relations (3.8) and (3.9) for the bound of dyonic mass hold good in $N = 4$ supersymmetric theory also.

4. Conclusions

In the supersymmetric Lagragian given by eq. (3.1) for dyonic system, the potential term $V(\phi, P)$ preserves the chiral symmetry but breaks the supersymmetry yielding the dyonic solutions given by eqs. (2.12) and (2.12a) under the conditions given by eq. (3.3). As such, the original massless gauge multiplets split into two different supersymmetric multiplets, one of which is still massless while the other one gets non-zero mass given by eq. (2.11). The mass formula given by eq. (3.9), or eq. (2.11) in saturated form, survives quantization by requiring that the supersymmetric charge algebra is saturated involving central charges which arise as surface terms when spontaneous symmetry breaking occurs.

The modification in supersymmetric algebra in the form given by eq. (3.10) leads to partial breaking of supersymmetry and the unbroken symmetry pairs the bosonic zero modes with fermionic zero modes. Eq. (3.17) shows that the dyonic configuration breaks only half of the supersymmetry of the $N = 2$ theory. Eq. (3.27) shows that half of the supersymmetries are destroyed by the dyonic solutions in $N = 4$ theory also. These eight destroyed supersymmetries satisfy a zero mode equation for fermionic fluctuations and for each fermionic zero mode there is bosonic zero mode (the correspondence is one to one [32]). The unbroken supersymmetry pairs bosonic zero modes with fermionic zero modes in $N = 4$ theory also. As such, the effective action governing the low energy dynamics of the dyons of $N = 4$ supersymmetry is given by $N = 8$ supersymmetric quantum mechanics based on the moduli space M_k. In $N = 4$ supersymmetric theory, the electrons and monopoles have same quantum numbers and hence the electric-magnetic duality is possessed contrary to $N = 2$ supersymmetric theory of dyons. It has recently been shown [33] that in the models of spontaneous breaking of $N = 2$ to $N = 1$ global supersymmetric theory and those of $N = 4$ to $N = 2$ global supersymmetric theory the parameters of electric and magnetic Fayet-Iliopoulos terms can be considered proportional to electric and magnetic charges of dyonic black-hole. As such the black-holes of these theories with one half of supersymmetry unbroken may be relevant to models of spontaneous breaking of $N = 2$ supersymmetry to $N = 1$. We shall take up this problem in details in our forth-coming paper.

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